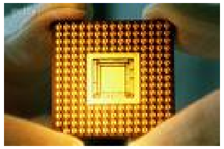
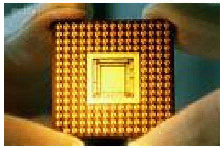


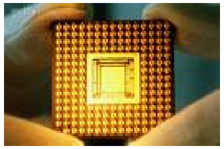
EE Modul 1: Electric Circuits Theory

- Basic Laws
- Circuit Theorems
- Methods of Network Analysis
- Non-Linear Devices and Simulation Models



EE Modul 1: Electric Circuits Theory

- Current, Voltage, Impedance
- Ohm’s Law, Kirchhoff's Law
- Circuit Theorems
- Methods of Network Analysis



Electric Charges

- Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

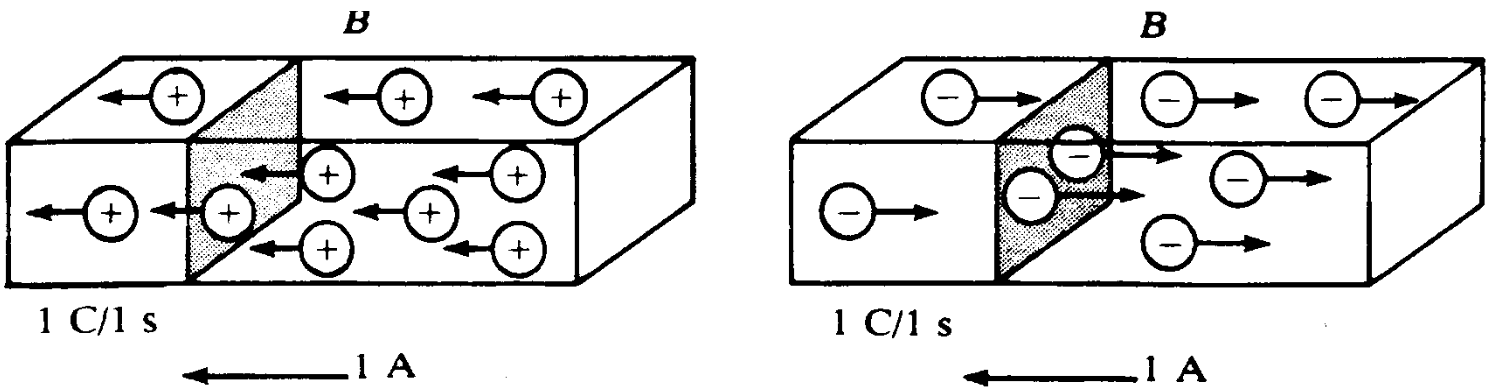
- The charge e on one electron is negative and equal in magnitude to 1.602×10^{-19} C which is called as electronic charge. The charges that occur in nature are integral multiples of the electronic charge.

Electric Current (1)

- Electric current $i = dq/dt$. The unit of ampere can be derived as $1\ A = 1 C/s$.
- A direct current (dc) is a current that remains constant with time.
- An alternating current (ac) is a current that varies sinusoidally with time.

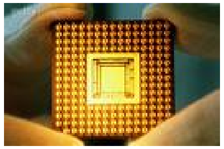
Electric Current (2)

The direction of current flow:



(a) (b)

Positive ions Negative ions



Electric Current (3)

Example 1

A conductor has a constant current of 5 A.

How many electrons pass a fixed point on the conductor in one minute?

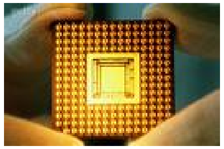
Solution

Total no. of charges pass in 1 min is given by

$$5\text{ A} = (5\text{ C/s})(60\text{ s/min}) = 300\text{ C/min}$$

Total no. of electrons pass in 1 min is given by

$$\frac{300\ \text{C/min}}{1.602x10^{-19}\ \text{C/electron}} = 1.87x10^{21}\ \text{electrons/min}$$



Electric Voltage

- Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

- Mathematically, $v_{ab} = dw / dq$ (volt)

  – w is energy in joules (J) and q is charge in coulomb (C).

- Electric voltage, v_{ab}, is always across the circuit element or between two points in a circuit.

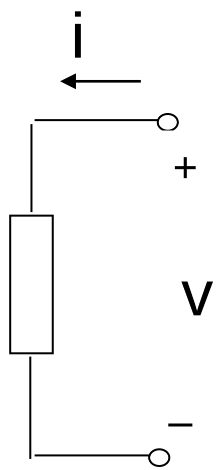
  $v_{ab} > 0$ means the potential of a is higher than potential of b.

  $v_{ab} < 0$ means the potential of a is lower than potential of b.

Power and Enegy (1)

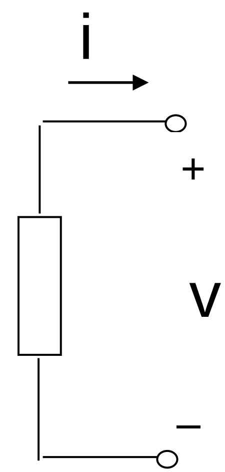
- Power is the time rate of expending or absorbing energy, measured in watts (W).

- Mathematical expression: $$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i$$



P = +vi
absorbing power

Passive sign convention



p = –vi
supplying power

Power and Enegy (2)

- The law of conservation of energy

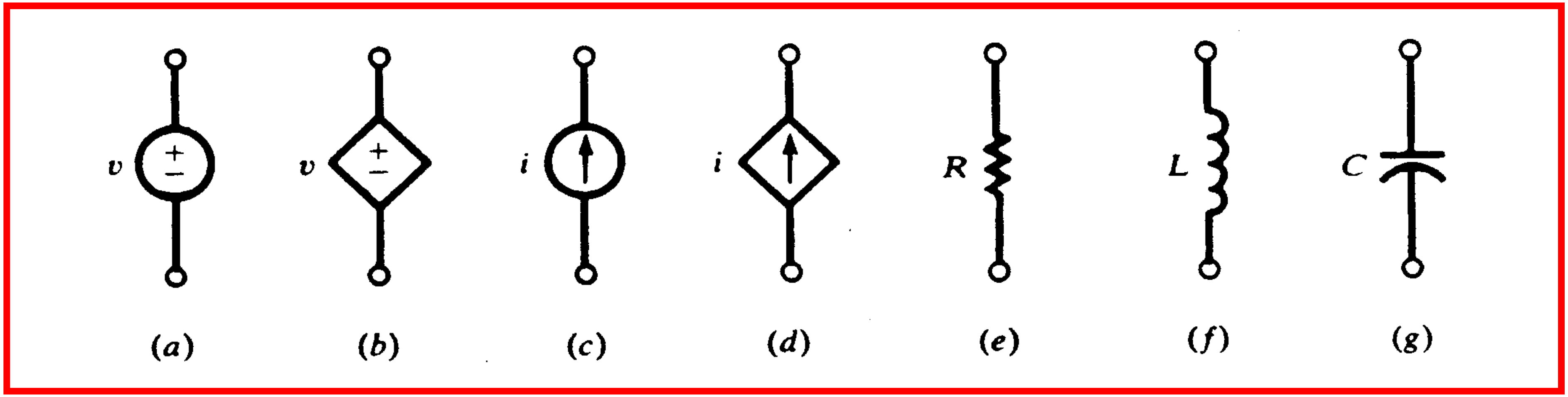
$$\sum p = 0$$

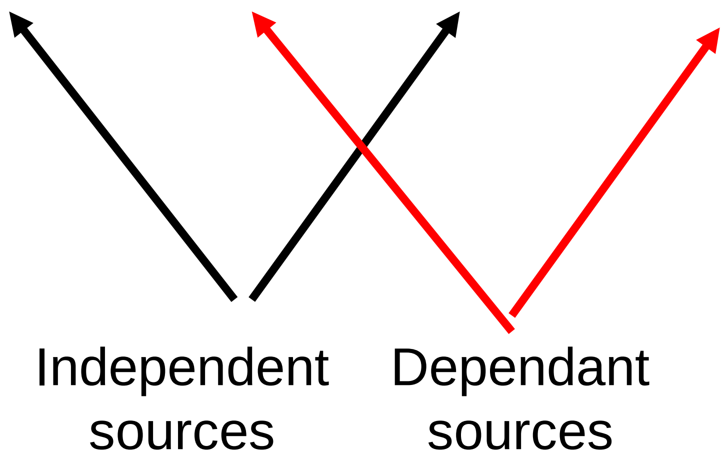
- Energy is the capacity to do work, measured in joules (J).
- Mathematical expression

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$$

Circuit Elements (1)

Active Elements | **Passive Elements**



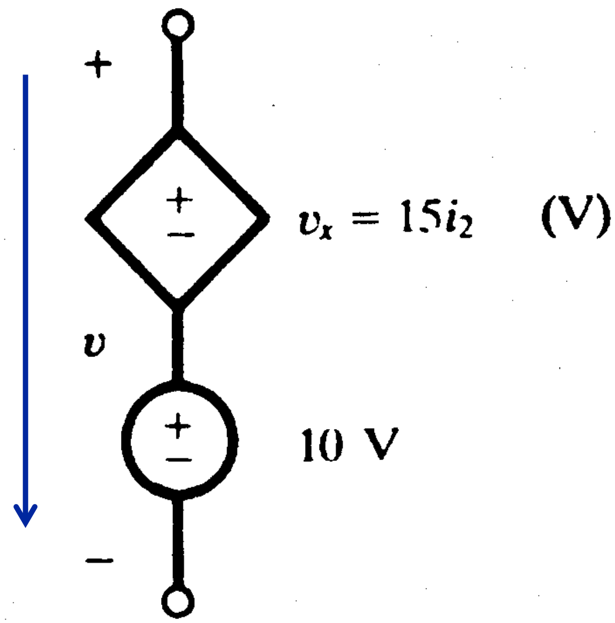


- **A dependent source is an active element in which the source quantity is controlled by another voltage or current.**
- **They have four different types: VCVS, CCVS, VCCS, CCCS. Keep in minds the signs of dependent sources.**

Circuit Elements (2)

Example

Obtain the voltage *v* in the branch shown below for i_2 = 1A.



Solution

Voltage *v* is the sum of the current-independent 10-V source and the current-dependent voltage source v_x.

Note that the factor 15 multiplying the control current carries the units Ω.

Therefore,

$v = 10 + v_x = 10 + 15(1) = 25$ V

BSC Modul 1: Electric Circuits Theory Basics

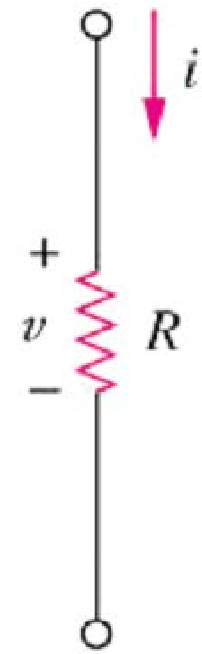
- Current, Voltage, Impedance
- **Ohm’s Law, Kirchhoff's Laws,**
- Circuit Theorems
- Methods of Network Analysis

Ohm‘s Law (1)

- Ohm’s law states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.
- Mathematical expression for Ohm’s Law is as follows:

$$v = i \cdot R$$

R = *Resistance*



- Two extreme possible values of R:

0 (zero) and ∞ (infinite)

are related with two basic circuit concepts:

short circuit and open circuit.

Ohm‘s Law (2)

- *Conductance* is the ability of an element to conduct electric current; it is the reciprocal of resistance R and is measured in siemens. (sometimes mho’s)

$$G = \frac{1}{R} = \frac{i}{v}$$

- The power dissipated by a resistor:

$$p = v \cdot i = i^2 \cdot R = \frac{v^2}{R}$$

Branches, Nodes, Loops (1)

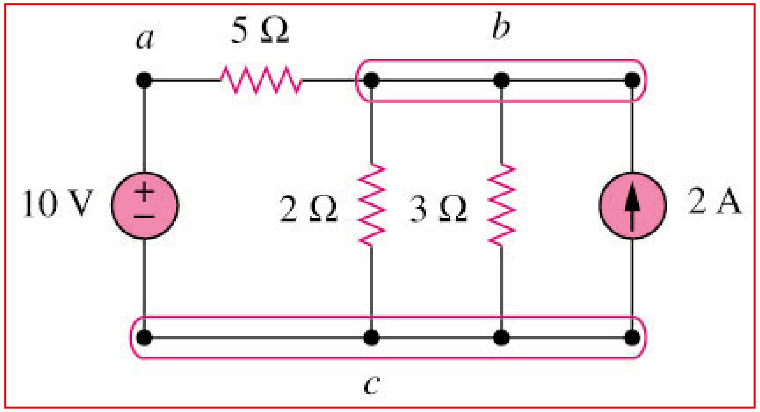
- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.

- A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

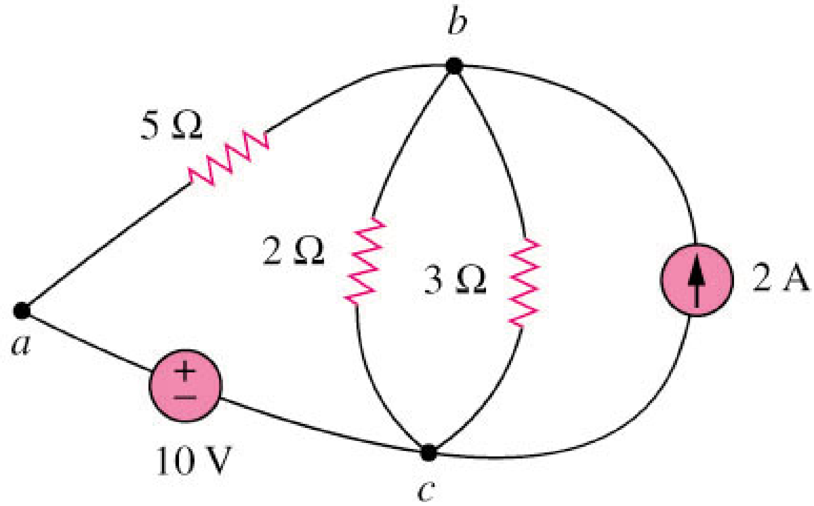
$$b = l + n - 1$$

Branches, Nodes, Loops (2)

Example 1



Original circuit



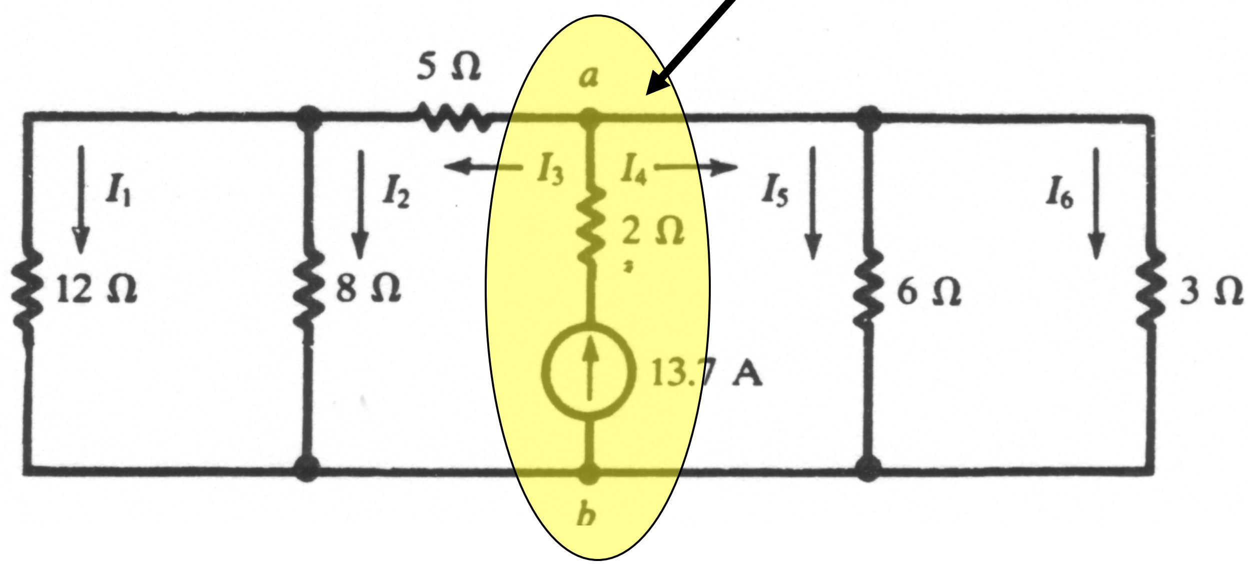
Network schematics or graph

How many branches, nodes and loops are there?

Branches, Nodes, Loops (3)

Example 2

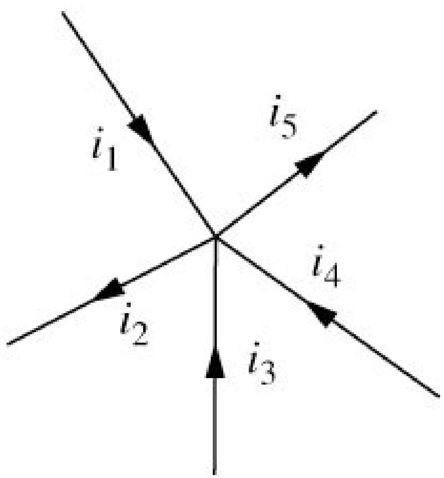
Should we consider it as one branch or two branches?

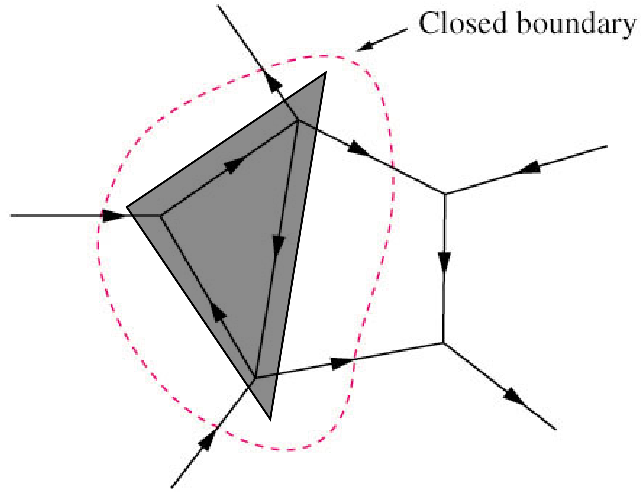


How many branches, nodes and loops are there?

Kirchhoff's Current Law (1)

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.



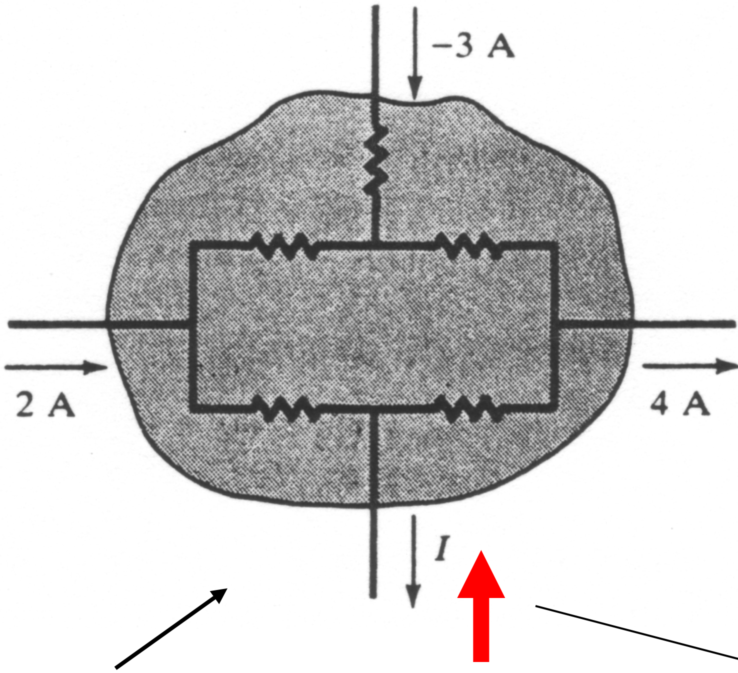


Mathematically,

$$\sum_{n=1}^{N} i_n = 0$$

Kirchhoff's Current Law (2)

- Determine the current I for the circuit shown in the figure below.



$I + 4 - (-3) - 2 = 0$

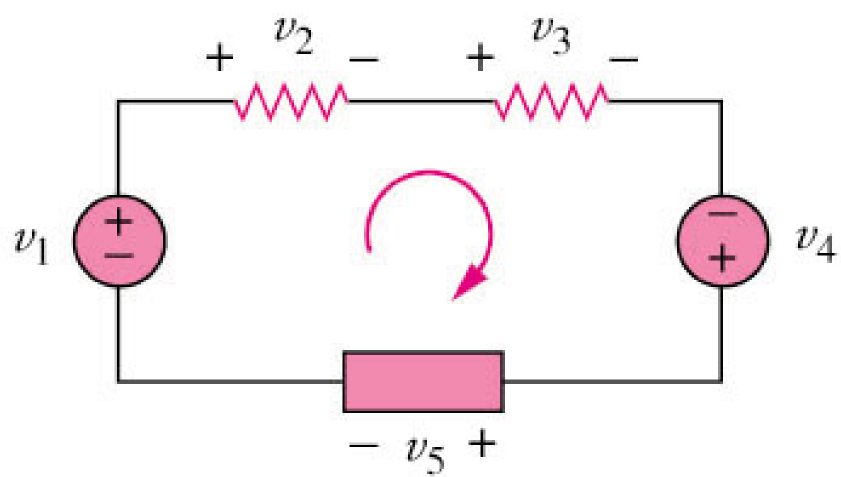
$\Rightarrow I = -5A$

This indicates that the actual current for I is flowing in the opposite direction.

We can consider the whole enclosed area as one "node".

Kirchhoff's Voltage Law (1)

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

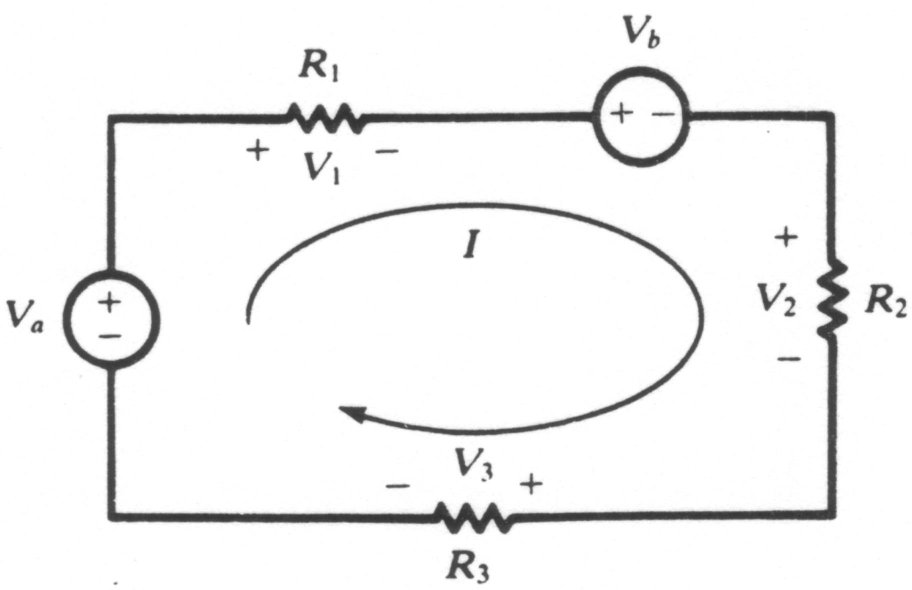


Mathematically, $$\sum_{m=1}^{M} v_n = 0$$

Kirchhoff's Voltage Law (2)

Example

- Applying the KVL equation for the circuit of the figure below.



$$v_a - v_1 - v_b - v_2 - v_3 = 0$$

$$v_1 = I \cdot R_1 ;\ v_2 = I \cdot R_2 ;\ v_3 = I \cdot R_3$$

$$\Rightarrow v_a - v_b = I \cdot (R_1 + R_2 + R_3)$$

$$\boxed{I = \frac{v_a - v_b}{R_1 + R_2 + R_3}}$$

Series Circuit and Voltage Division (1)

- Series: Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.
- The equivalent resistance of any number of resistors connected in a series is the sum of the individual resistances.

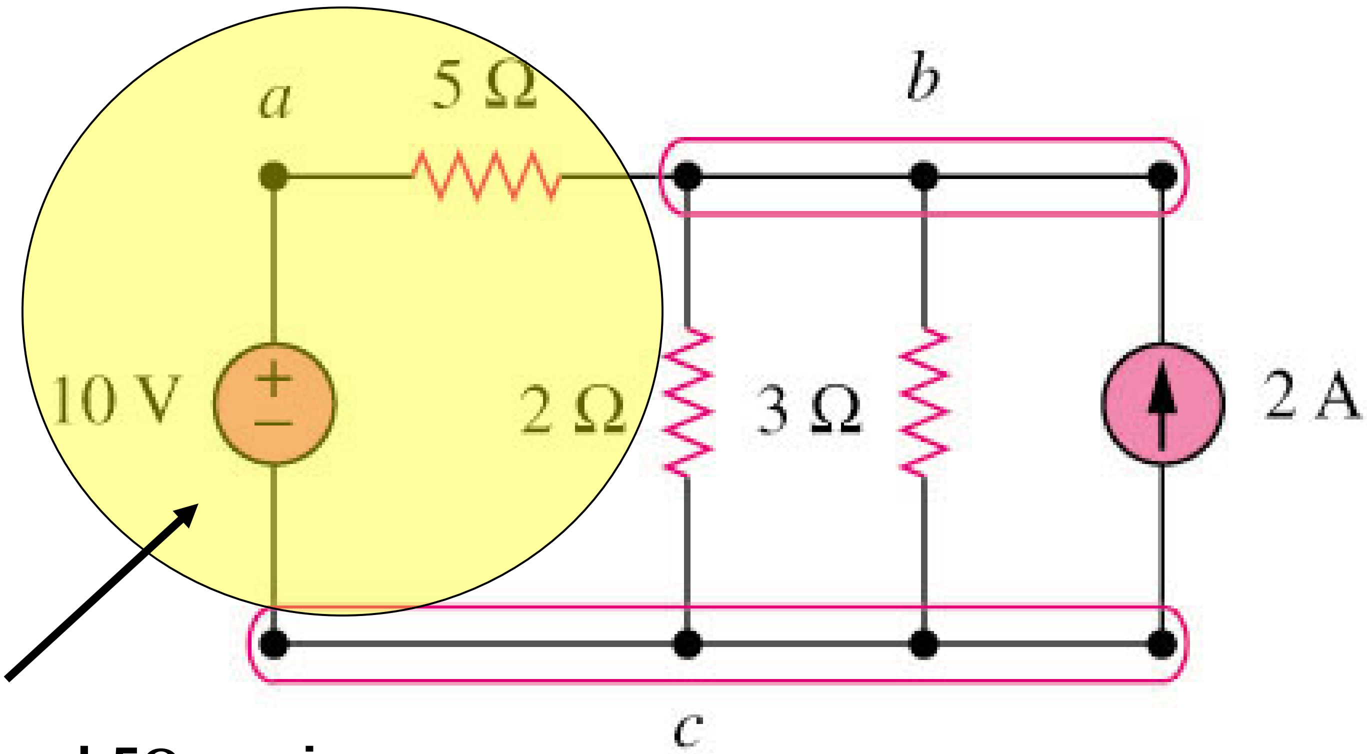
$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

- The voltage divider can be expressed as

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Series Circuit and Voltage Division (2)

Example



10V and 5Ω are in series.

Parallel Circuit and Current Division (1)

- Parallel: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

- The equivalent resistance of a circuit with N resistors in parallel is:

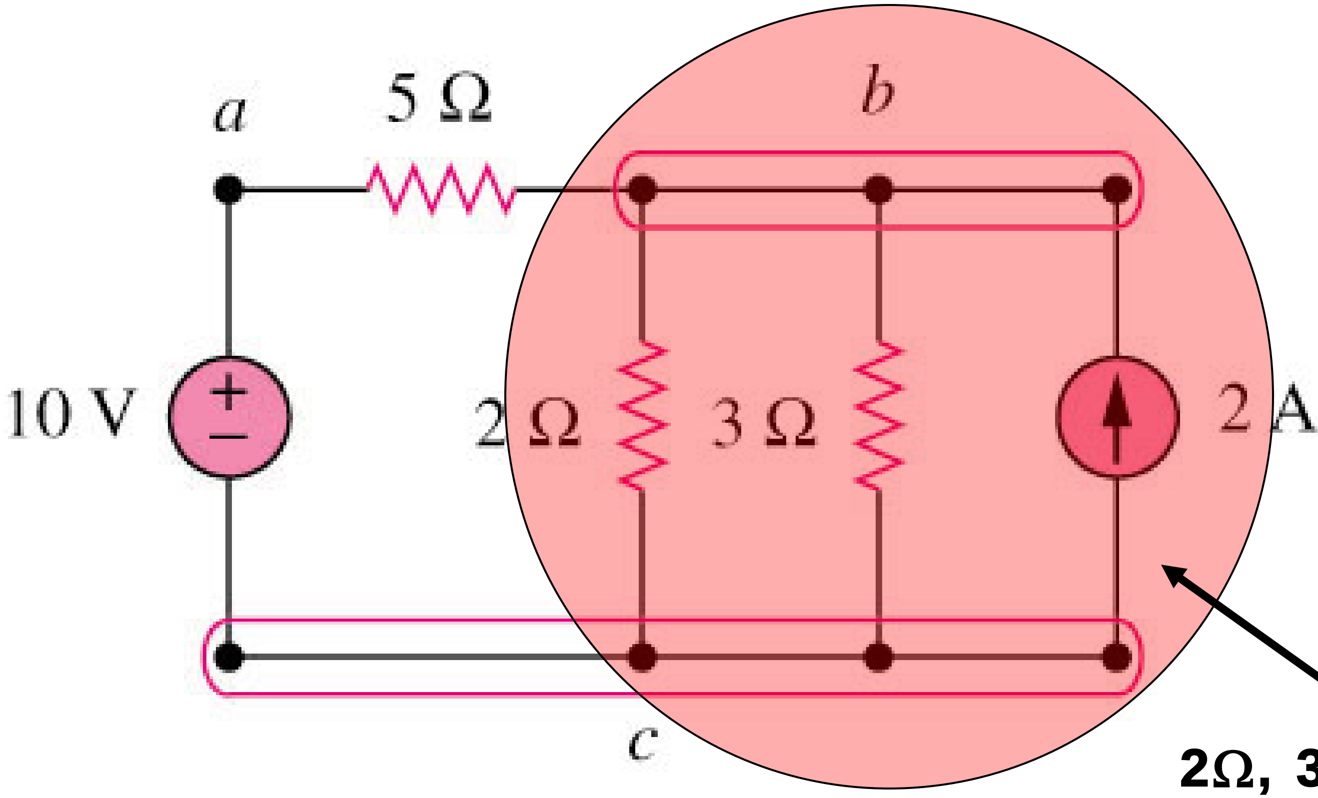
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- The total current i is shared by the resistors in inverse proportion to their resistances. The current divider can be expressed as:

$$i_n = \frac{v}{R_n} = \frac{i \cdot R_{eq}}{R_n}$$

Parallel Circuit and Current Division (2)

Example



2Ω, 3Ω and 2A are in parallel

BSC Modul 1: Electric Circuits Theory Basics

- Current, Voltage, Impedance
- Ohm’s Law, Kirchhoff's Laws,
- **Circuit Theorems**
- Methods of Network Analysis

Linearity Property (1)

It is the property of an element describing a linear relationship between cause and effect.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Homogeneity (scaling) property

$$v = i\,R \quad \longrightarrow \quad k\,v = k\,i\,R$$

Additive property

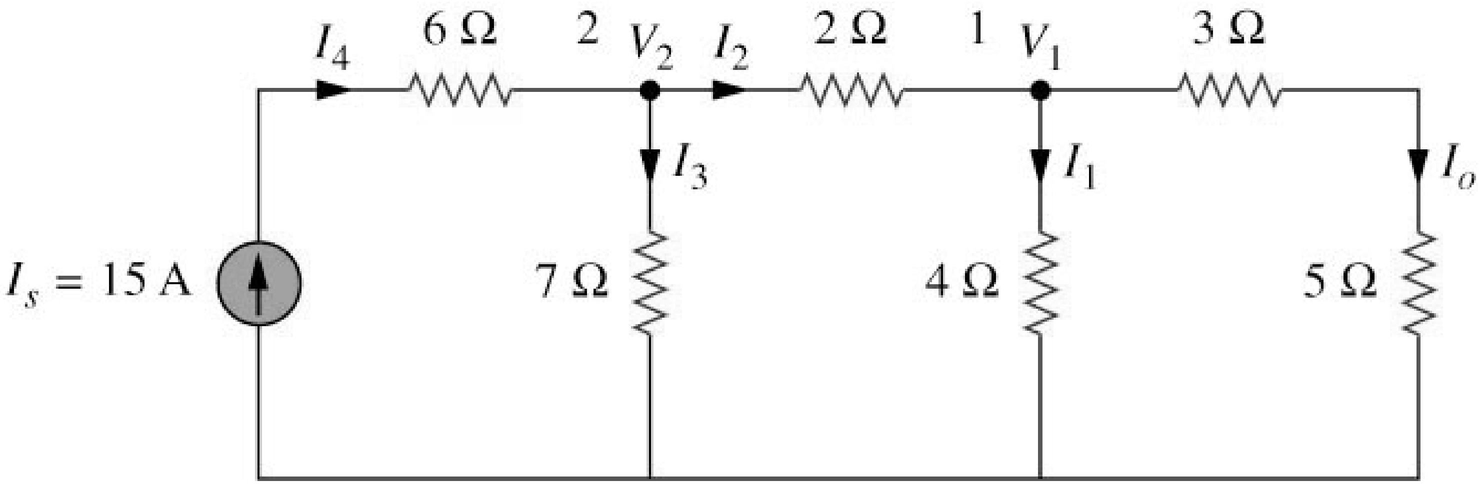
$$v_1 = i_1\,R \text{ and } v_2 = i_2\,R$$

$$\longrightarrow \quad v = (i_1 + i_2)\,R = v_1 + v_2$$

Linearity Property (2)

Example

By assume I_o = 1 A for I_S = 5 A, use linearity to find the actual value of I_o in the circuit shown below.



Answer: I_o = 3A

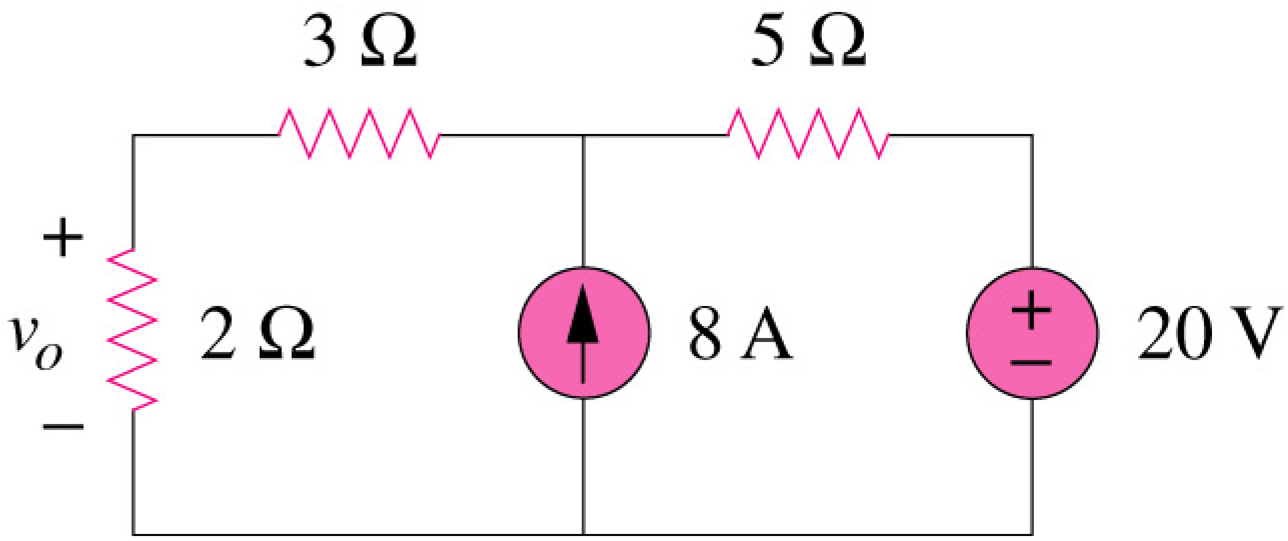
Superposition Theorem (1)

It states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

Superposition Theorem (2)

We consider the effects of the 8A and 20V sources one by one, then add the two effects together for final v_o.



Superposition Theorem (3)

Steps to apply superposition principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Superposition Theorem (4)

Two things have to be keep in mind:

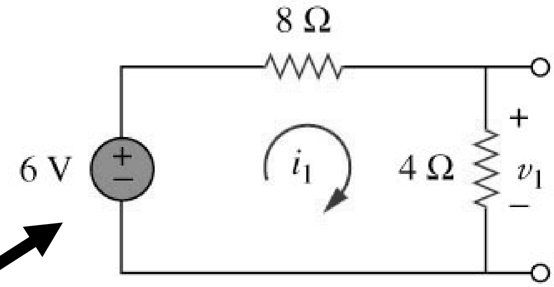
1. When we say turn off all other independent sources:
   - Independent voltage sources are replaced by 0 V (short circuit) and
   - Independent current sources are replaced by 0 A (open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables.

Superposition Theorem (5)

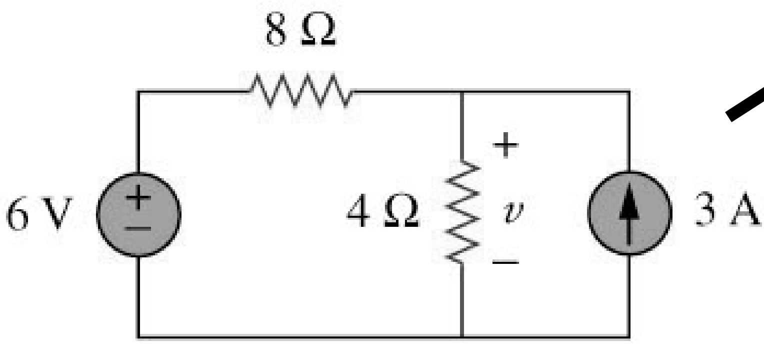
Example

Use the superposition theorem to find v in the circuit shown below.

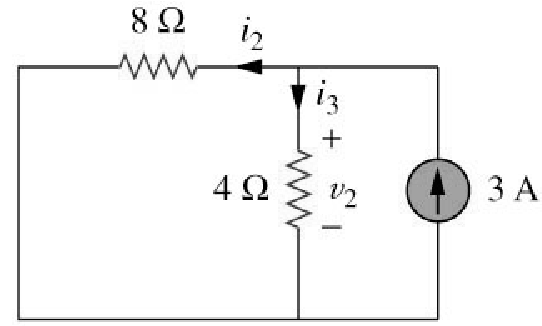


(a)

3A is discarded by open-circuit



6V is discarded by short-circuit



(b)

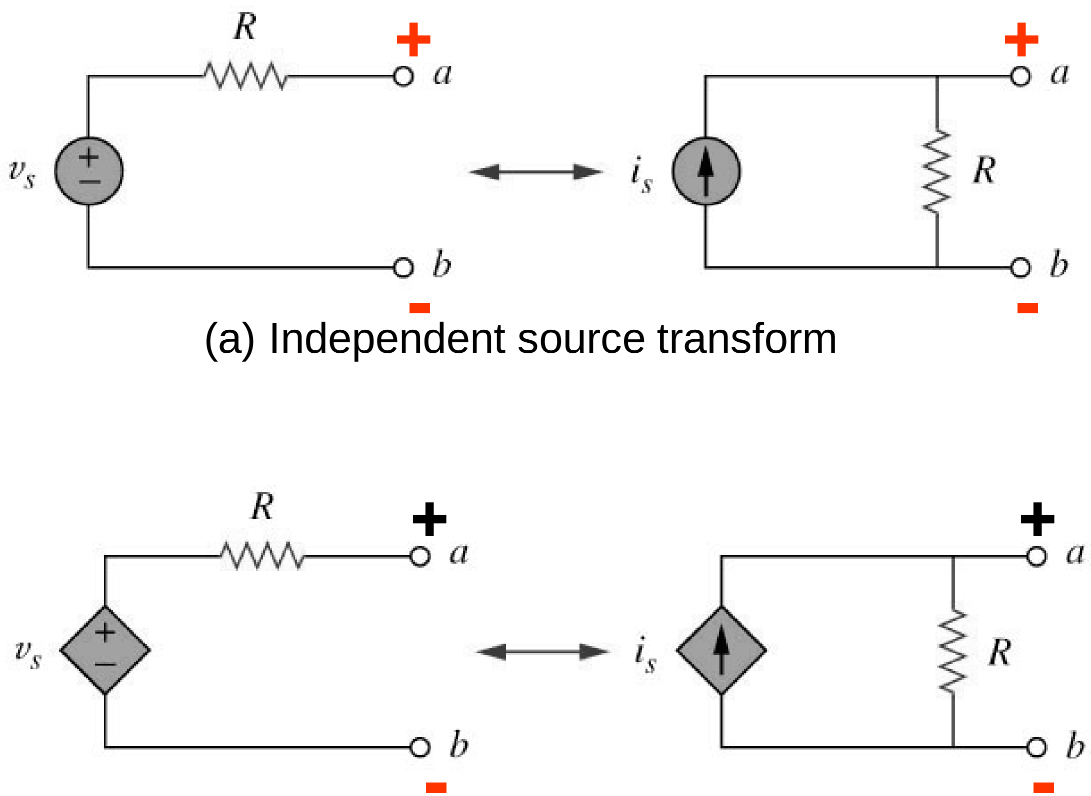
Answer: v = 10V

Source Transformation (1)

- An equivalent circuit is one whose *v-i* characteristics are identical with the original circuit.

- It is the process of replacing a voltage source v_S in series with a resistor R by a current source i_S in parallel with a resistor R, or vice versa.

Source Transformation (2)



(a) Independent source transform

(b) Dependent source transform

$$R = \frac{v_s}{i_s}$$

v_s open circuit voltage

i_s short circuit current

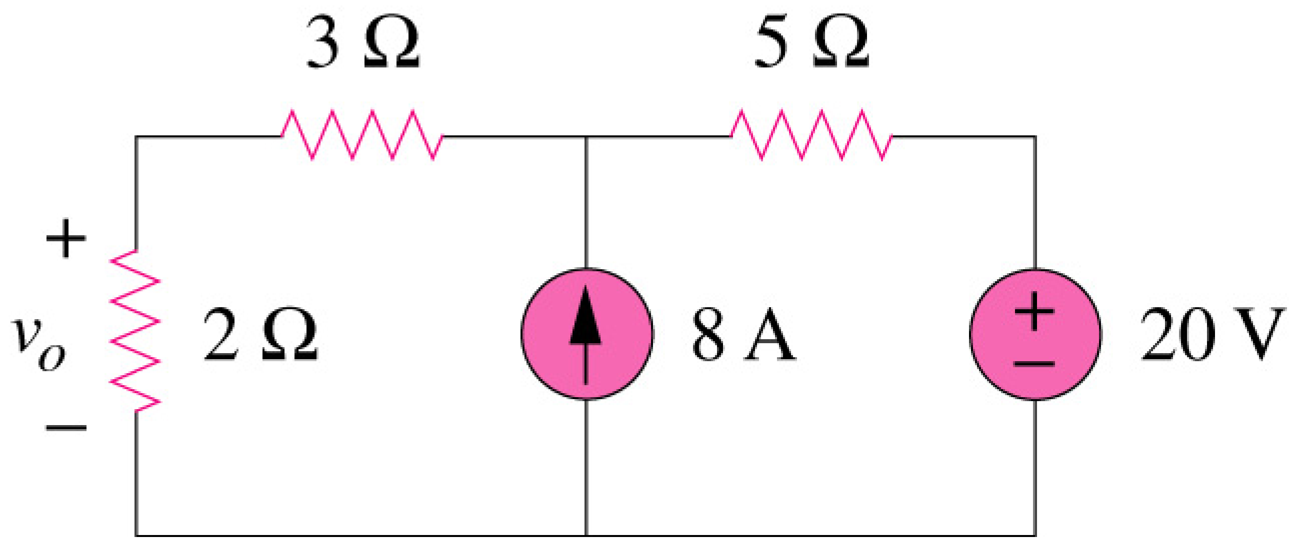
Remarks:

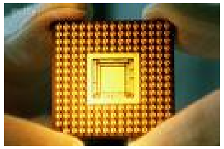
- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

Source Transformation (3)

Example

Find v_o in the circuit shown below using source transformation.





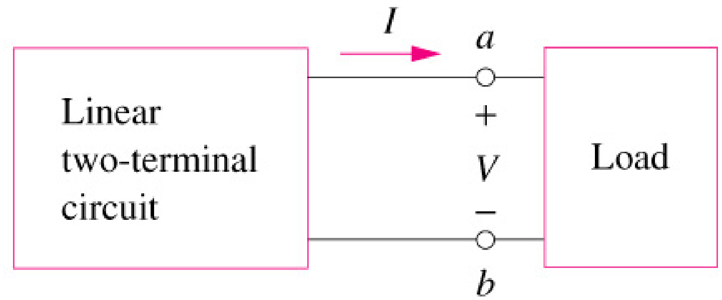
Thevenin's Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source V_{TH} in series with a resistor R_{TH},

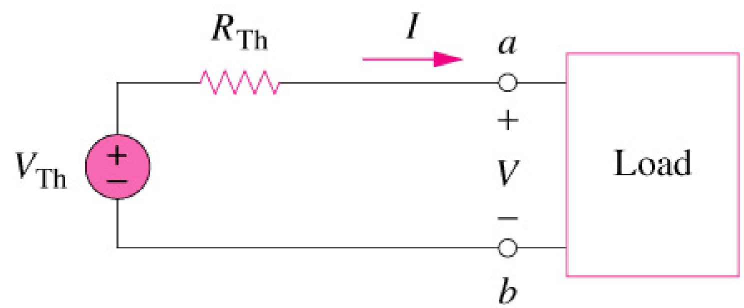
where

- V_{Th} is the open-circuit voltage at the terminals.

- R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



(a)

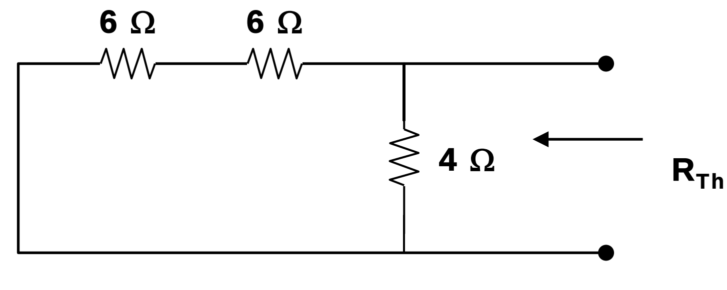


(b)

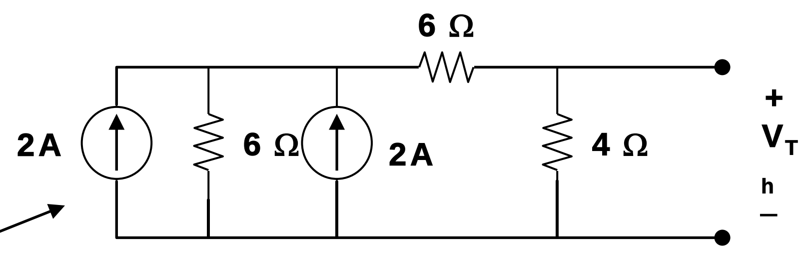
Thevenin's Theorem (2)

<u>Example</u>

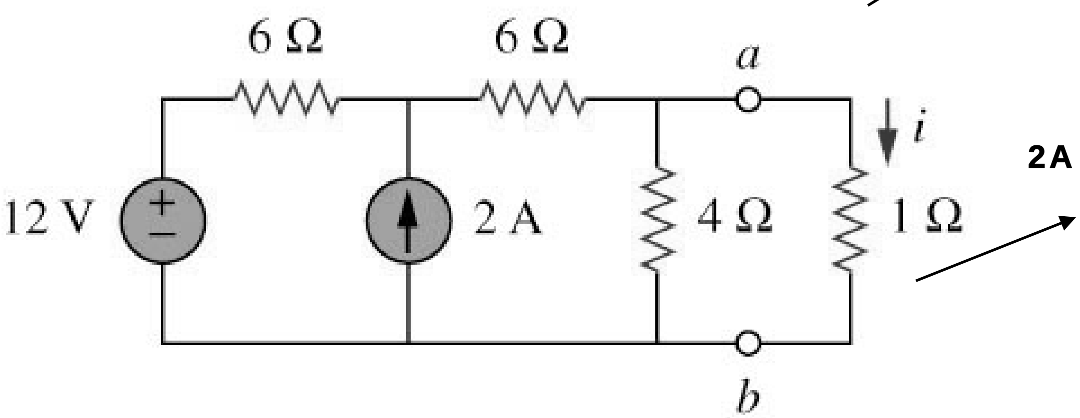
Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find i.



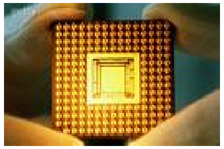
(a)



(b)



Answer: V_{TH} = 6V, R_{TH} = 3Ω, i = 1.5A

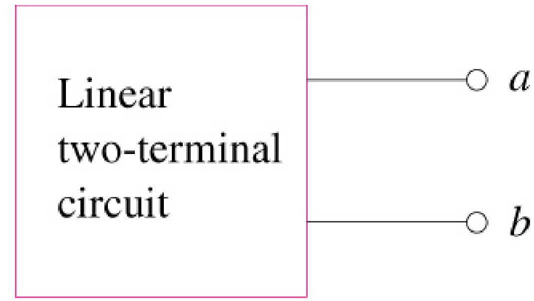


Norton's Theorem (1)

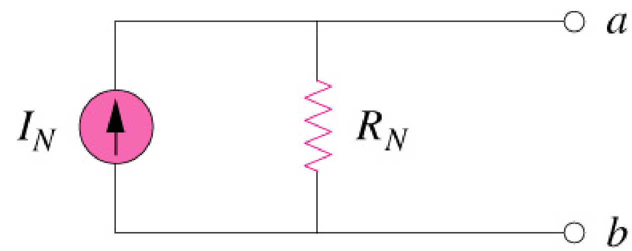
It states that a linear two-terminal circuit can be replaced by an equivalent circuit of a current source I_N in parallel with a resistor R_N,

Where

- I_N is the short circuit current through the terminals.
- R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

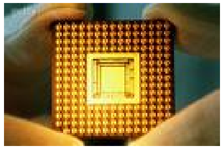


(a)



(b)

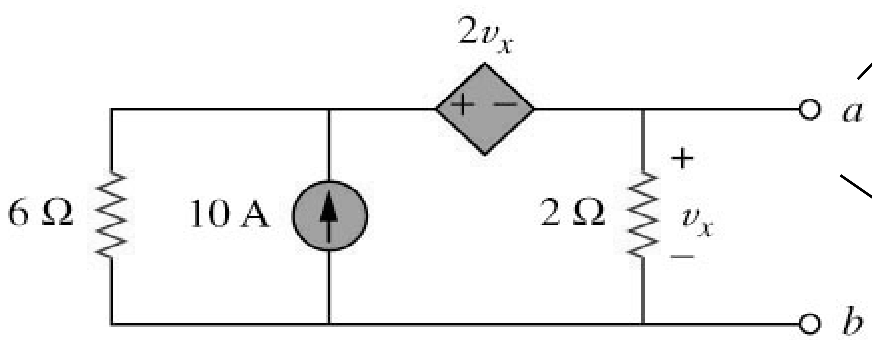
The Thevenin's and Norton equivalent circuits are related by a source transformation.

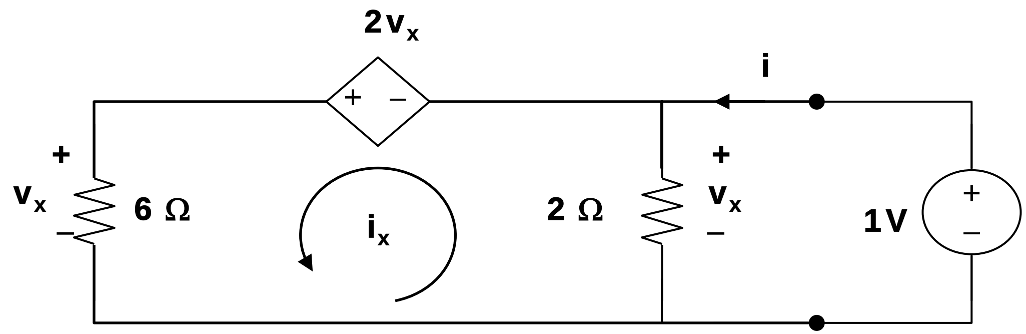


Norton's Theorem (2)

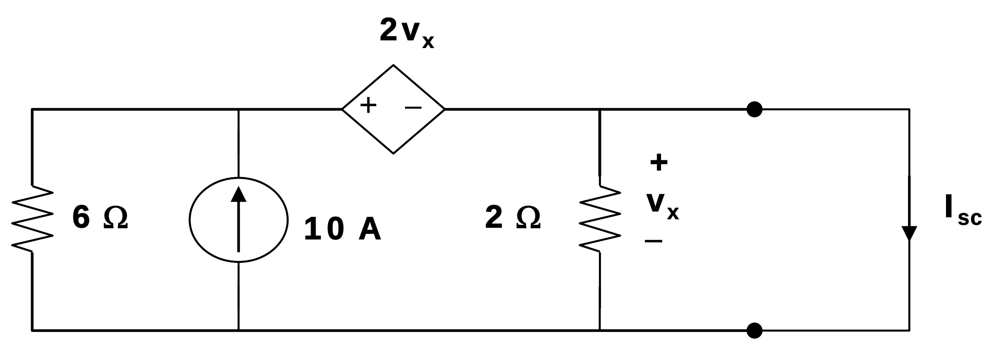
Example

Find the Norton equivalent circuit of the circuit shown below.



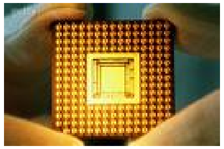


(a)



(b)

Answer: $R_N = 1\Omega,\ I_N = 10A$



EE Modul 1: Electric Circuits Theory

- Current, Voltage, Impedance
- Ohm’s Law, Kirchhoff's Laws
- Circuit Theorems
- **Methods of Network Analysis**

Introduction

Things we need to know in solving any resistive circuit with current and voltage sources only:

Number of equations

- Ohm's Law — b
- Kirchhoff's Current Laws (KCL) — n-1
- Kirchhoff's Voltage Laws (KVL) — b – (n-1)

mesh = independend loop

Number of branch currents and branch voltages = 2b (variables)

Problem: Number of equations!

Mesh Analysis (1)

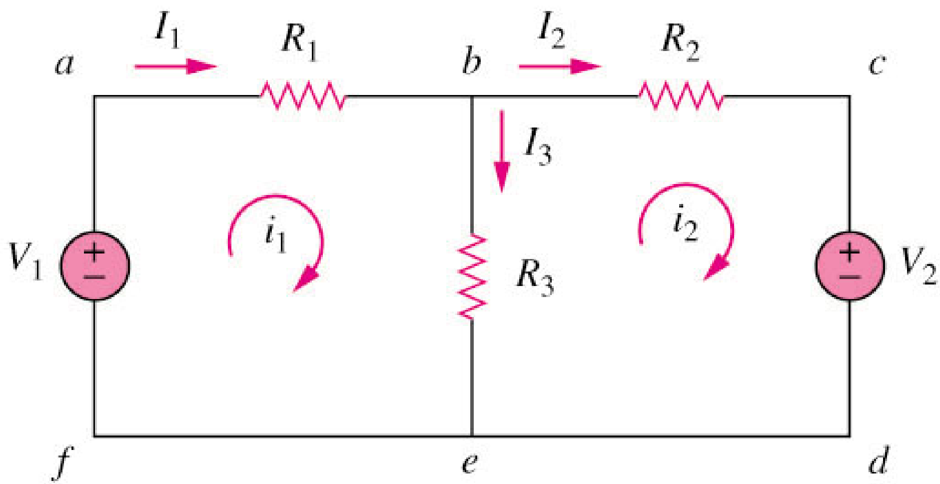
1. Mesh analysis provides a general procedure for analyzing circuits using <u>mesh currents</u> as the circuit variables.

2. <u>Mesh analysis applies KVL</u> to find unknown currents.

3. A <u>mesh</u> is a loop which does not contain any other loops within it (independent loop).

Mesh Analysis (2)

Example – circuit with independent voltage sources



Equations:

$$R_1 \cdot i_1 + (i_1 - i_2) \cdot R_3 = V_1$$
$$R_2 \cdot i_2 + R_3 \cdot (i_2 - i_1) = -V_2$$

reordered:

$$(R_1 + R_3) \cdot i_1 - i_2 \cdot R_3 = V_1$$
$$- R_3 \cdot i_1 + (R_2 + R_3) \cdot i_2 = -V_2$$

Note:

i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

Mesh Analysis (3)

Formalization: Network equations by inspection.

$$\begin{pmatrix} (R_1 + R_3) & -R_3 \\ -R_3 & (R_2 + R_3) \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ -V_2 \end{pmatrix}$$

Impedance matrix Excitation

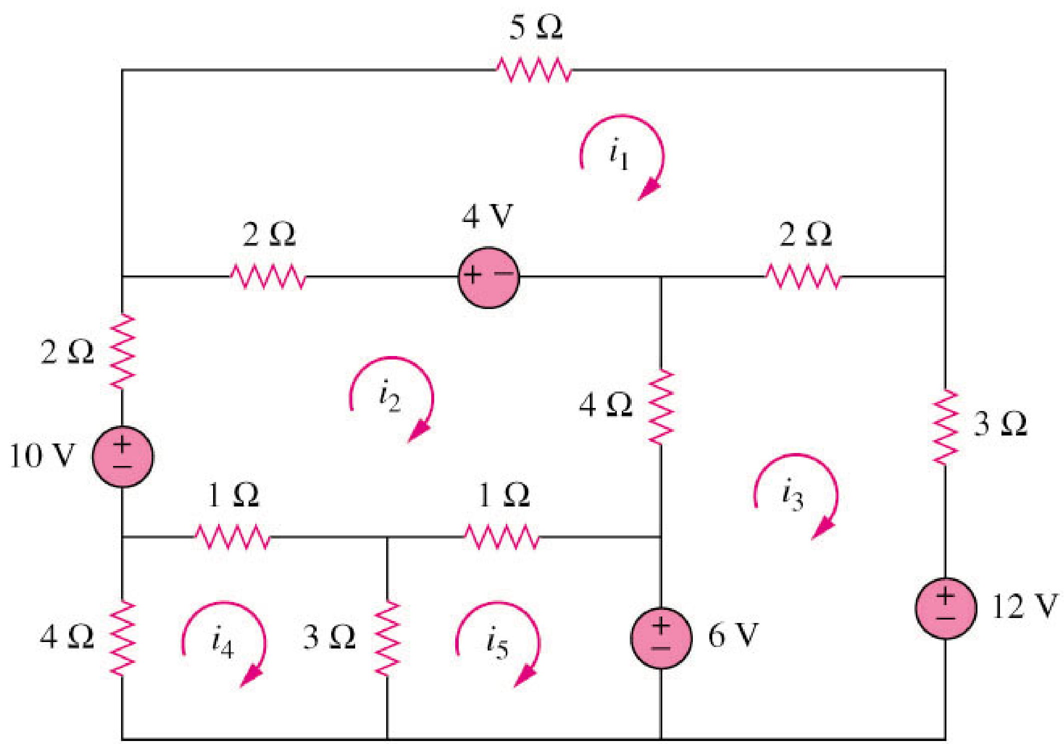
Mesh currents

General rules:

1. Main diagonal: ring resistance of mesh n
2. Other elements: connection resistance between meshes n and m
    - Sign depends on direction of mesh currents!

Mesh Analysis (4)

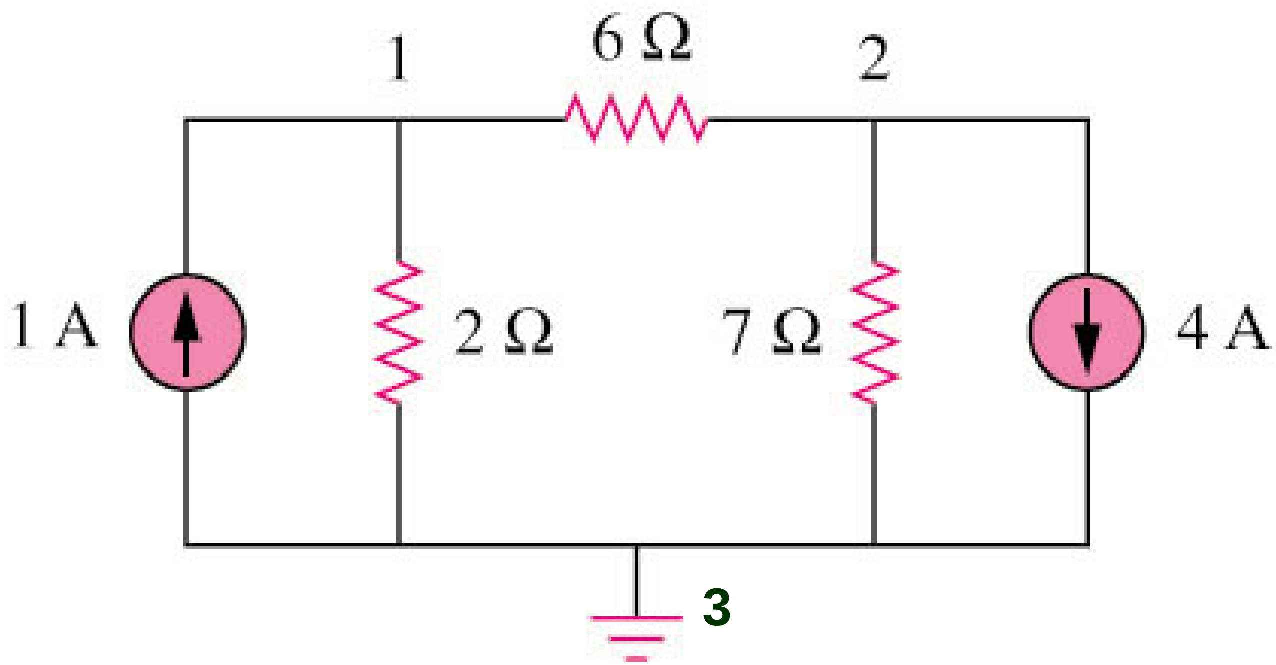
<u>Example:</u> By inspection, write the mesh-current equations in matrix form for the circuit below.



Nodal Analysis (1)

It provides a general procedure for analyzing circuits using node voltages as the circuit variables.

Example



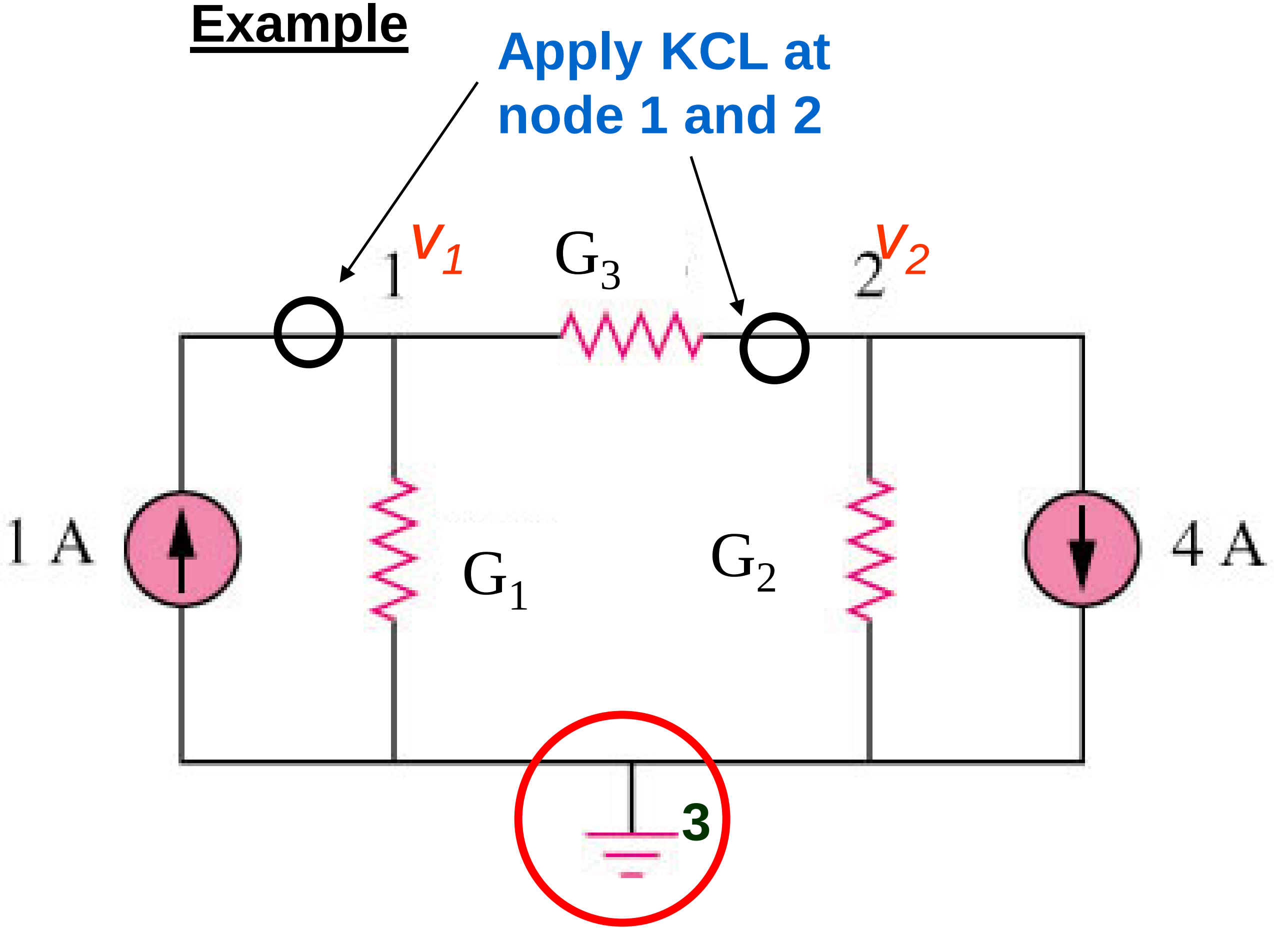
Nodal Analysis (2)

Steps to determine the node voltages:

1. Select a node as the reference node.
2. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Nodal Analysis (3)

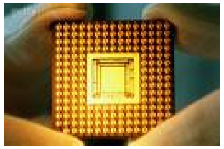
Example



$G_1 \cdot v_1 + (v_1 - v_2) \cdot G_3 = 1A$
$G_2 \cdot v_2 + G_3 \cdot (v_2 - v_1) = - 4A$

reordered:

$(G_1 + G_3) \cdot v_1 - v_2 \cdot G_3 = 1A$
$- G_3 \cdot v_1 + (G_2 + G_3) \cdot v_2 = - 4A$



Nodal Analysis (4)

Formalization: Network equations by inspection.

$$\begin{pmatrix} (G_1 + G_3) & -G_3 \\ -G_3 & (G_2 + G_3) \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1\mathrm{A} \\ -2\mathrm{A} \end{pmatrix}$$

Admittance matrix　　　　Excitation

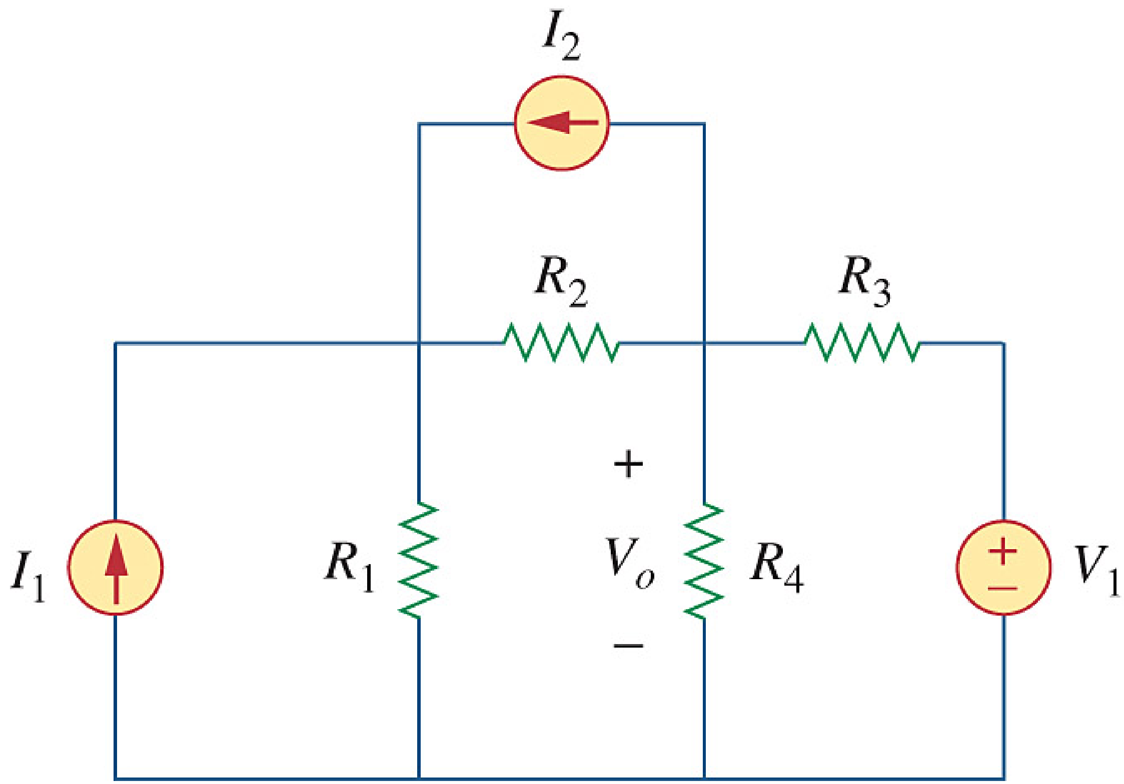
Node voltages

General rules:

1. Main diagonal: sum of connected admittances at node n
2. Other elements: connection admittances between nodes n and m
   - Sign: negative!

Nodal Analysis (5)

Example: By inspection, write the node-voltage equations in matrix form for the circuit below.



Summary